Published in the Proceedings of Mind II conference, Dublin, Ireland, September 1997

Creativity and the neural basis of qualia.

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Abstract

In what computational aspect is the brain different from the computer? In what objective measures can the brain said to be “creative”? These are the fundamental questions that concerns the neural basis of human mental activity. Here we discuss several important aspects of the essential computational ingredients of human mind in order to understand the “creative” process going on in the brain. One of the key concepts is the nature of the source of "externality" that adds new ingredients to the system and its output. We argue that in addition to information input and stochasticity, we need to consider a third possibility, namely "dynamics-embedded externality". We discuss how the neural origin of the subjective sensory qualities (qualia) is related to this aspect of creativity. The invariance of qualia under a certain class of transformation, and the mapping of discrete, incomplete relations between neural firings to the continuous, complete functional relations that seem to underlie mental phenomena are important elements in such a picture. We propose that we consider seriously the possibility that the "dynamics-embedded externality" introduced by qualia underlies human creativity.

1. Introduction.

At present, there are already several things that a computer does better than a human. For most numerical tasks, computers outdo the humans. However, in pattern recognition humans are by far better than computers yet. The gap between humans and computers increase rapidly as we move onto such highly cognitive tasks as language comprehension, and the gap seems insurmountable when we move on to the domain of creative works such as literature and art. It is often argued that even a mouse hitting the keyboard can, given a sufficient time, produce the entire work of Shakespeare. But we are yet to witness a computer producing a single convincingly creative work (without, of course, the aid of a human artist manipulating the input devise!). To put it shortly, the computer have not even started being creative. What is the essential difference between the human mind and the computer such that the former is creative and the latter is not?

Penrose put forward the conjecture that a Turing machine cannot create any work of art (Penrose 1989, 1994). That is because the computer does not have the *understanding* of what is involved. Penrose then goes on to say in order to *understand*, we need to have *awareness*. Subjectively, Penrose's conjecture seems reasonable enough. We need to be aware of what we are doing when we are creative! The really difficult question, however, is the following: In what objective terms can the human brain be said to *understand* or to be *aware*? When trying to answer these questions, we believe that we need to focus on the issue of the neural basis of *qualia*. When we are aware, we have qualia. Our ability to understand something seems to be closely related to the occurrence of qualia in perception. Here, we are using the word qualia in a generalised sense, which includes the meaning of words, locus in the visual field, and the more abstract concepts. In general, qualia in this sense makes it possible for several perceptual elements to be processed in a parallel and yet integrated manner in our mind. If qualia are central in our conscious mental activities, it logically follows that we need to look into the neural basis of qualia in order to understand the computational process that goes on in our brain.

In this paper, we argue that the question whether a brain is more than a computer or not is related to the nature of the neural origin of qualia. The question of qualia, how difficult its elucidation might be (Chalmers 1995), should be central to our understanding of the computational processes in the brain. Our perception is constructed *in terms of* qualia. What does having qualia have to do with the computational process in the brain? Why do we need to have qualia at all? We start to investigate these questions by looking at the role played by "externality" in the creative process.

2. What provides the Externality?

In any attempt to emulate creativity artificially, we make use of a system (e.g. digital computer). The system gives an output, and when the output shows some novel features which fits certain criteria (beauty, truth, usefulness, etc.), we say that the system is creative. So being creative needs to involve two things. Novelty and constraints (which constrains the system in such a way that the output fits the required criteria. It is an extremely interesting question what the exact nature of the constraints are, but at this moment we shall not go into its details.) In what follows, we call the source of novelty the *externality*, since it is something that is external to what is already in the system.

Formally, externality can be defined as follows. In any dynamical system, we have the temporal evolution of the system

$$\Omega(\tau) \to \Omega(\tau + d\tau)$$

where τ is the time parameter appropriate for describing the system. In such a case, it is possible to have a one to one mapping

$$\Omega(\tau_1) \to \Omega(\tau_2)$$

where $\tau_2 > \tau_1$

We define externality as a situation where the above one to one mapping is no longer possible. For an open system, there are two logical possibilities for the source of externality. Namely, (1) input from outside the system and (2) stochasticity. When these elements are present, it is no longer possible to have a one to one mapping from $\Omega(\tau_1)$ to $\Omega(\tau_2)$

A system described by quantum mechanics is one with such an externality. Namely, it is commonly believed that the stochasticity in quantum mechanics is of a random nature (Omnes 1994), and it is impossible to derive a unique state $\Omega(\tau_2)$ given $\Omega(\tau_1)$ where $\tau_2 > \tau_1$. (There is one possible twist here. Penrose argues that the wave function reduction process might be a non-computable, but *deterministic* process, providing any system that implements it with the ability to go beyond Turing machines. It is Penrose's conjecture that the human brain implements this aspect of quantum mechanics at the level of microtubules. If Penrose's conjecture is correct, then we do have a unique state $\Omega(\tau_2)$ given $\Omega(\tau_1)$, although the transition is in general non-computable).

It should be noted that externality introduced by the input of information from the environment is unlikely to play an essential role in the creative process *per se*. A trivial example is the function of an image processor where the original image is created by a human artist and put into the system (the image processor in this case) through a scanner. The input of information from the environment can provide a material, but unless there is a genuinely creative process (which provides the novelty without decaying out of the constraints) going on in the system, the system cannot be said to be creative.

Stochasticity, on the other hand, is not likely to provide any meaningful externality to the system either. In the self-organizing systems (e.g development from an egg to a chick), where form is seemingly created *de nuvo*, stochasticity seems to provide the variation from the prototypical form, not the primal cause for the generation of the prototype itself. It is known that the fractal growth of DLA (diffusion limited aggregation) can be realized within a purely deterministic dynamics (see Sander *et al.* 1985 for example). The deterministic prototype possesses all the essential features we usually associate with a DLA, such as the radiating bifurcation structure. In biological developments, biochemical or biophysical fluctuations are likely to provide the necessary directionality (in terms of free energy gradient) to the overall reaction of the system. However, the stochasticity itself is not the primary cause for the generation of forms itself. This is evident in the role thermal fluctuation plays in the structure of biochemical reaction network. The structure is determined by the "geometry" of space spanned by the reaction rates, and stochasticity introduced by fluctuations does not affect it (e.g. Mogi 1993). In biological development, therefore, the generation of form itself seems to be emulatable by a deterministic process, as is the case with deterministic DLA.

Our tentative conclusion at this stage is that neither (1) input from outside the system nor (2) stochasticity inherent in the dynamics is likely to provide the necessary externality to sustain creativity as is manifested by the human brain.

Then, what is providing the externality that underlies human creativity? We conjecture that the externality as is evident in the creative process should be provided in a form *embedded* in the dynamics of the system (Fig.1). Namely, we conjecture that there is a certain aspect of the dynamical law that dictates the way the system evolves which provides the necessary externality to sustain the creative process. Since neither the input of information from outside the system nor the stochasticity inherent in the dynamics is likely to provide the externality necessary for sustaining the novelty involved in the creative process, we believe that externality embedded in the dynamics of the system is the only logical possibility, although admittedly its exact nature is not at all clear at present.

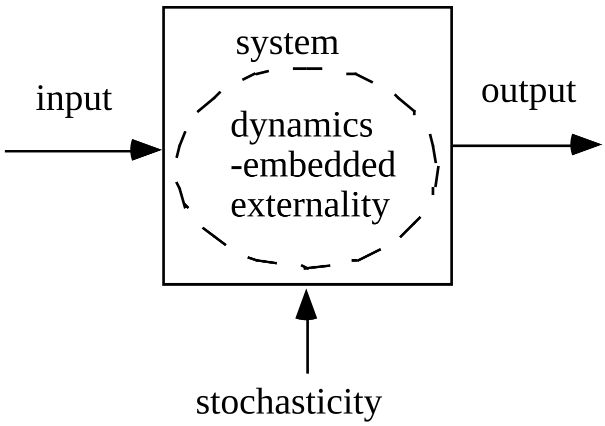


Fig. 1 Externality as the source of creativity

3. Mach's principle in perception

In the following, we argue that the "dynamics-embedded" externality put forward in the previous section is closely related to the neural basis of human mental activities. Since our conjecture is that the externality which sustains a creative process should be provided embedded in the dynamics of the system, it follows that in the case of human creativity, the externality should be provided as one embedded in the dynamics of neural networks in the brain.

In the following argument, we discuss the possibility that qualia, the subjective qualities of our senses, provide the necessary externality to the neural networks. In order to put our conjecture in perspective, we need to go back to the foundations of the neural correlates of perception.

Given the knowledge of neurophysiology today, it is reasonable to assume that any aspect of our mental phenomena is ultimately explainable in terms of the neural firings in our brain (the neuron doctrine in perception). The neuron doctrine was first put forward by Barlow (1971) in a now classic paper. We can phrase the neuron doctrine as we understand it today as follows.

Our perception is directly invoked by the neural firings in the brain. A non-firing neuron is as good as non-existent as far as perception is concerned. The characteristics of our perception at one psychological moment should be explained by the nature of neural firings at that psychological moment only.

The requirement that our perception should be explained in terms of the properties of neural firings only is a strong one than is usually realized. Indeed, a more radical rephrasing of the neuron doctrine, Mach's Principle in perception (Mogi 1997), demonstrates the inherent radicalism behind the neuron doctrine.

In perception, the significance of a firing neuron is determined by its relation to other firing neurons at that psychological moment.

At any psychological moment, it is reasonable to assume that our perception is based on the properties of neural firings in our brain at *that* psychological moment only. A rather blunt way to put this is that the only information available for a subject to construct its mind is the information on the state of the neural firings in his or her brain. If we consider any specific firing neuron in the network, its significance in our perception, whatever it may be, should be prescribed by the relation between the neural firing in question and other firing neurons in the network. For example, if a neuron codes for the perception of a "rose", it does so not because it responds selectively to the presentation of a rose as the stimulus, but because the neuron in question is endowed with that particular significance in perception through its interactions with other firing neurons in the network. Namely, we come to the important conclusion that we cannot take the selective responsiveness of neurons as the foundation for the neural correlates of perception.

If we adopt Mach's principle in perception, the significance of a firing neuron can be established only by considering the relation between the firing neurons as is specified by the interaction via the synapses. Under such a picture, a percept should be defined as an interaction-connected cluster of neural firings, not as a single neural firing or a group of neural firings selectively responsive to a particular feature in the environment.

This line of thought is related to a new trend in cognitive science which is attracting a growing attention, namely radical constructivism (von Glaserfeld 1995). Namely, it is customary to assume that in perception we represent something that is already out there in the environment. However, such a framework is incompatible with the neuron doctrine or its radical counterpart Mach's principle in perception. According to a radical constructivist's view, the essence of what perception is all about is more aptly understood in terms of the German word *Vorstellung*. When Kant's "Critique of Pure Reason" was translated into English, *Vorstellung* was misleadingly translated as *representation*. The German word indicates a range of things that take place spontaneously in someone's head and do not require an original. In contrast, the English word "representation" normally refers to a set of stimuli that more or less reliably evokes a certain range of neural responses (von Glaserfeld 1997). The very concept of response selectivity is based on the premise of representing something that is already out there in the environment. We argue that we should start from the mutual relations between the neural firings instead, as is in line with Mach's principle in perception. Namely, we should treat perception not as representation, but as *Vorstellung*. It is in this context that the neural basis of qualia should now be considered.

4. Invariance of qualia under transformation of neural firings.

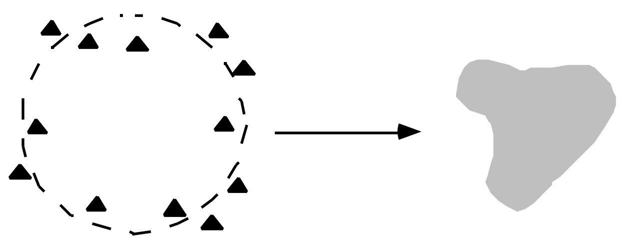
The origin and nature of qualia, the subjective sensory qualities accompanying our perception, is generally regarded as the most hard of the problems concerning the relation between the mind and the brain (Crick

1994, Chalmers 1995). Although it will be years before we come to any understanding of this intriguing aspect of our mind, and therefore nature, it is likely that without incorporating qualia, we cannot really grasp the essence of what our perception is all about.

Under Mach's principle, properties of qualia should be deduced from the causal relationships between the firing of cortical neurons that underlie our perception. For example, the quality of the "redness" of "red" does not originate in the physical properties of the pattern of wavelength of photons falling on the retina. The quality of the "redness" should be deduced from the mutual relationship between the neural firings that underlie colour perception, within the context of colour constancy (Land 1983) if necessary. Namely, a certain quale is generated in our mind when a certain (ultimately mathematical) relation between neural firings is realized. For example, "redness" of "red" will be invoked when the neural firings in brain areas including area V4 satisfy a certain mutual relationship, which is to be expressed in some yet unclarified mathematical form.

An interesting problem arises here about the neural basis of qualia. The neural firings in the brain are discrete and full of noise. Any causal relationship between neural firings is therefore expected to be discrete and full of noise. The qualia that emerges in our mind, on the other hand, appears to possess what some authors call "features of the Platonic world" (e.g. Penrose 1989), namely qualities that seemingly reflect complete mathematical relationships. The interesting question is, how does the brain "interpolate" the discrete and incomplete relationship between neural firings so that our mind can get hold of the complete mathematical relationships that underlie our perception?

For the purpose of a graphic depiction of the relation between qualia and the neural firings in the brain, let us imagine that a certain quale is invoked only when the neural firings are arranged in a "circle" in some abstract space. Since the neural firings always suffer from noise, we can never have a perfect circle. However, it is reasonable to consider that the quale that is invoked in our mind would be based on the "perfect" mathematical construct of a circle (Fig.2). Our mind should therefore be able to "interpolate" the discrete neural firings and somehow form the perfect circle from the discrete, imperfect collection of neural firings.



neural firing pattern qualia

Fig.2

We have qualia in our perception only when we are conscious. One of the functions of consciousness therefore would be to interpolate the incomplete causal relationships between neural firings (which underlie our perception according to Mach's principle). Under this picture, consciousness exists in order to map the discrete and incomplete causal relationships into the complete, mathematical relationships.

The above interpolation process can be generalised to the idea of invariance involved in the neural basis of qualia. Specifically, it is expected that given a certain spatio-temporal firing pattern P, we have the same qualia in our mind for an equivalence class $\{P\}$, which is defined by a certain group of transformations. For example, as long as the same firing patterns are reproduced in our brain, it is expected that qualia invoked by the neural firings are the same no matter when and where our brain is located. (The "redness" of "red" seems to be the same no matter where and when we perceive it.) Namely, the qualia induced by neural firings will be invariant under translation in time (T), translation in space (S), and rotation in space (R). Going a step further, it is not unreasonable to assume that qualia induced by neural firings is invariant when the set of neural firings invoking them undergoes Lorentz transformation. There are a number of interesting mathematical problems yet not fully addressed here. In considering the neural basis of qualia, therefore, it is crucially important to study the invariance of qualia under a set of transformations exerted on the neural firing patterns. Such an invariance is expected to be important in understanding the origin and nature of qualia.

The postulated interpolation involved in the correspondence between the discrete, incomplete relationship among neural firings and the complete mathematical relationships that underlie qualia can now be understood as part of the more general scheme of the invariance of subjective sensory qualities under certain transformations exerted on the neural firings. Namely, the interpolation which maps a range of neural firing patterns to the same qualia in our mind is nothing else than the invariance of qualia under a group of transformations between neural firing patterns.

This feature of the neural basis of qualia is important not only in considering how the mind arises from the neural activities in the brain, but also in considering the neural basis of creativity. We come back to this issue in section 6 after reviewing one important aspect of human creativity in the next section.

5. Creativity should be considered as a class

A universal Turing machine can perform any computation. But there are things that are considered to be out of the realms of what it can do, and human creative process may be one of them. However, at present it is not at all clear whether human creativity actually is out of the realms of what a universal Turing machine can in principle emulate. In order to evaluate whether human creative process is beyond the realm of what a Turing machine can do, let us review what kind of argument is used to prove that there are some specific tasks that a Turing machine cannot do.

We number the Turing machines as T_n, and write the output of the *n*th Turing machine when provided with an input of *m* as $T_n(m)$
, where *n* and *m* are natural numbers. There is a universal Turing machine, which can perform the computation of the *n*th Turing machine if provided with the initial input of *n*. We do not consider here what significant difference the limitations on the physical memory space and computation time makes on the computational ability of the Turing machines. We can prove that there is at least one computation that a Universal Turing machine cannot do, namely to determine whether the *n*th Turing machine, when provided with the input *m*, will halt or not. A program that successfully judges whether a Turing machine halts or not is nothing else than a function $h(n, m)$ which takes the value of either 0 or 1 in such a way that when $h(n,m)=1$ the computation halts, and when $h(n,m)=0$, it does not. It can be proved, using Cantor's diagonal slash argument, that there is no single Turing machine that can implement this particular function *h*. Therefore, there is at least one function (namely the function *h*) that a Turing machine cannot implement.

We note here that the above argument considers the halting problem as a class. Namely, we are asking whether there is an algorithm that successfully solves the halting problem for the whole set of Turing machines T_n $(n=1, 2, 3,....)$, and we get a negative answer. However, if we ask instead whether there is an algorithm which successfully determines whether a specific computation halts or not, the answer can in general be yes. Thus, we note one important aspect of the issue of computability. The question of computability should always be considered as a class.

In general, if we take any specific input and specific output, it is always possible to find an algorithm that successfully emulates that particular input-output relation. Consider language comprehension, for example. The Turing test (Turing 1950) is widely regarded as a legitimate test whether the intelligence of a computer is equivalent to the human or not. Let us consider a specific conversation between two humans A and B, and try to emulate the part played by B with a computer. It is always possible to program the computer in such a way that the response of the computer is indistinguishable from its human counterpart B. Of course, in order to do so, we need to know what A is going to say beforehand! We can do that as long as we are dealing with *a specific* example of conversation. This is cheating, because the difficulty of producing a program that passes the Turing test consists in the fact that human conversation is often unpredictable, and we cannot know beforehand what A is going to say.

The above argument demonstrates that the ability to comprehend language should be considered in terms of a class. Given a particular example of a conversation, it is always possible to program the computer in such a way that that particular example of conversation is reproduced. It is the ability of humans to be able to carry out conversation as a class, namely to be able to respond to any conversation within a certain class (i.e. within a set of meaningful remarkable) that is so hard for any computer produced so far to emulate (see Chomsky 1986 for related arguments).

In a similar manner, creativity should be considered in terms of a class. It is always possible for a computer to produce a particular work of art, if we know beforehand what that particular work is going to be. However, this is not creativity *par excellence*. It is the ability of the artist to create works of art as a class that is the remarkable achievement of human intelligence. As we have seen in section 2, a creative work should satisfy two conditions, novelty and constraints. It is the ability of an artist to produce works of art that consistently satisfy the constraints that is at the heart of the issue of human creativity. Therefore, whenever we put forward the question whether an artificial system can emulate human creativity or not, we need to ask the question in the context of a class. It is not enough for a computer to produce a single work of art that is interesting. It is the ability to *consistently* create works that satisfy the constraints that is so hard to emulate by any artificial system so far.

We conclude this section thus. Whether a Turing machine can be programmed to be creative or not, an issue which is not settled yet, is likely to involve an argument similar to the halting problem. We need to consider human creativity in the context of a class. A Turing machine can never be programmed to consistently tell whether a computation is going to halt or not. Likewise, the real issue is whether a Turing machine can be made to be consistently creative or not, a question that is at present still open.

6. Qualia as a source of externality in creative process

Now we come to our central argument. Namely, we discuss the possibility that the qualia that accompany our mental activities provide the necessary externality that sustains human creativity.

Since every mental phenomena is underlain by neural firings, human creativity is to be ultimately based in the dynamics of neural activities. We have concluded above that the externality that underlies creativity should be embedded in the dynamics of the system. Neither the information input from the environment nor the stochasticity inherent in the dynamics are likely to provide the externality necessary to sustain the creative process. Then how is the externality embedded in the dynamics of neural networks?

As we discussed in section 4, a process of interpolation is likely to be involved in the correspondence between qualia in our mental activities and neural firings in the brain. An interesting question concerning the role of qualia in our mental activities is whether they are just epiphenomena or play a more active role. It has proved difficult to envisage any active role for qualia without entering into the realm of naked dualism (Eccles 1990) . We argue here that one interesting possibility for the role of qualia in human mental activity is that it affects the dynamics of neural networks through the interpolation process. A unique feature of this possibility is that we may then be able to discuss the role played by qualia in human mental activity in a mathematically well-defined manner so that we can build a quantitative model, thus making it possible to test any hypothesis that we may come up with experimentally.

At the end of the day, qualia are just epiphenomena unless by their presence in our mental activity, *the objective behaviour of the neural network in our brain is affected*. Specifically, we put forward the conjecture that neural networks might follow a dynamical rule in such a way that the neural firings are reflected in the behaviour of the system *as if* they satisfy the interpolated complete mathematical relationships which underlie the qualia in our mental activities. The objective behaviour of the neural networks will be different because of the "interpolation" process in such a picture.

Since under Mach's principle in perception the neural correlates of qualia are the interaction-connected cluster of neural firings, it follows that some non-local features of the neural network is involved. Namely, we propose to consider the following possibility.

There are some aspects of neural network dynamics that is invoked only when a certain non-local feature of neural firing is activated. This feature provides the necessary "externality" that underlies human creative process through an interpolation process. That non-local feature of neural firing gives rise to qualia in our perception at the same time.

There are some subtleties here, especially concerning the meaning of the statement that we can no longer have a one to one mapping

$$\Omega(\tau_1) \rightarrow \Omega(\tau_2)$$

where

$$\tau_2 > \tau_1$$

.

If the interpolation process can be given in a deterministic way, we may actually be able to give a one to one mapping if we take into account the interpolation process as well. However, it will be in general impossible to describe the temporal evolution of the system in terms of a reductionistic law, because in order to arrive at the interpolation we need to consider the mutual relations between a multitude of neural firings, and then feed them back into the basic dynamical law itself.

We believe that this is a very interesting possibility. In particular, we have seen that in any model of creative process, the system must be made to be able to produce creative works (i.e. both novel and satisfying certain constraints) in a consistent manner. If the correspondence between the neural firings in the brain and the qualia that are invoked in our mind are realized through some kind of interpolation process, that interpolation process might be able to provide externality to the system (in this case the neural network) in such a way that the necessary constraints are consistently satisfied. Since qualia are likely to represent what are meaningful to the human mind (and since human creativity is evaluated according to these standards as is prescribed by what are meaningful to the human mind), it is not at all implausible that the externality (if any) introduced by qualia to the dynamics of neural network will be of such a nature that it can sustain human creativity whose hallmark is the high rate of successful works which satisfy the required constraints.

7. Conclusion.

In this paper, we have put forward the argument that the neural basis of qualia is likely to be deeply involved in the human creative process. We have argued that the invariance under a group of transformations, the interpolation process in particular, is likely to provide the necessary externality which is needed to sustain a creative process.

One of the difficulties in designing an artificial system that implement creativity is that it is difficult to make the system produce novel features which consistently satisfy the required constraints. One idea that have been put forward is that the novelty might be provided by a kind of random generation process, followed by a filtering process which selects only these outputs that satisfy the requirements. Although we might be able to come up with some useful system with such a scheme, it is not likely that the human creative process itself is actually realized through such a scheme. It would appear that the "success rate" of a creative individual is too high for such a scheme to be working behind his or her mind. Such a difficulty will not be encountered if the novel features are introduced by the externality accompanying the interpolation process which generates qualia.

Although the present scheme is admittedly speculative at this stage, we believe that the difficulty encountered in understanding the neural basis of human creativity justifies at least considering such a possibility seriously.

Acknowledgements

This work was supported in part by Human Frontiers Fellowship grant to K.M.

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