Constrained Approximate Pattern Matching And Its Applications

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Abstract

Let $\mathcal{S} = \{S_1, ..., S_n\}$ and $\mathcal{Q} = \{Q_1, ..., Q_m\}$ be two point sets in the Euclidean d-dimensional space $\mathbb{E}^d$. We consider the problem of finding a transformation T of the space of transformations $\mathcal{T}$ so that when applied to point set $\mathcal{S}$ we maximize a matching of $\{T(\mathcal{S}), \mathcal{Q}\}$. We design randomized algorithms that constrain the pattern matching by setting geometric restrictions on T and extend the approach to geometric hashing. The time complexity of the algorithms is sensitive to the self-similarity of point sets and relies on the efficiency of nonlinear range query data-structures. We present applications of the techniques in vision geometry for image registrations and its relatives.

1 Introduction.

We consider pattern matching problems where point sets $\mathcal{S}$ and $\mathcal{Q}$ may have different cardinalities and where point coordinates are inaccurate (for example, provided by a corner detection algorithm). Therefore we have to somehow identify the *inliers* (points that match) from the *outliers*. Measuring the quality of a matching given a transformation T can be computed by a percentile-based Hausdorff distance which allows to discard the effect of k outliers: $h_k(\{\mathcal{S}, \mathcal{Q}\}) = \min_{k\,S \in \mathcal{S}} \min_{Q \in \mathcal{Q}} d(S, Q)$, where $\min_k$ returns the smallest k-th element and $d(\cdot, \cdot)$ is a distance function. However a drawback of the Hausdorff measure is that a point can be "matched" several times which often conveys no meaning in problems arising in vision geometry (see bottleneck matching algorithms for an alternative). We refer the reader to the survey of Alt and Guibas [1] for a state of the art of point pattern matching. Irani and Raghavan [2] described a randomized Monte-Carlo approach for finding a transformation provided that sets $\mathcal{S}$ and $\mathcal{Q}$ match at least densely, i.e. at least a portion of the points. Another approach in practice to pattern matching is to consider the space of parameters defining the space of transformation matrices $\mathcal{T}$ and apply the branch-and-bound algorithm [3, 4] on those generating parameters. For example, allowing translations and rotations on the plane defines 3x3 matrices (using homogeneous coordinates) generated by three parameters.

2 Randomized Constrained Alignment Method.

Let k be the number of pairs of points required to define a transformation $T \in \mathcal{T}$ (for example $k = 2$ for translation, rotation and uniform scaling factor). If we assume that α percent of the points match, then with probability roughly $1 - \alpha^k$ by taking a subset of k points from $\mathcal{S}$ and testing it with all other k-subsets of $\mathcal{Q}$, we obtain a satisfying transformation [2] (within some error tolerance). Moreover if we want to test for a k-subset if the transformation let match α percent of

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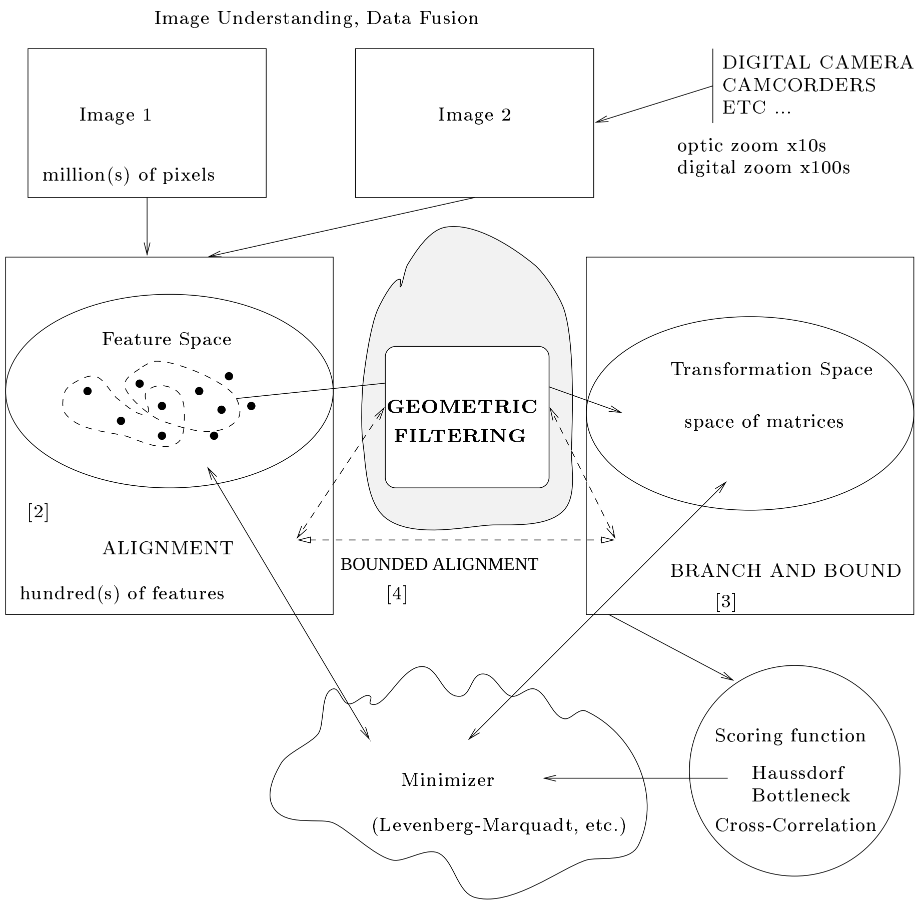
the points then we can first test it for a subset of size r. Then if that subset matches for at least β percent then we test the all set, otherwise we can reject that transformation (eventually discarding a good transformation with small probability). Repeating $O(1)$ times the above scheme let us get an $O_{\alpha,\beta}(k!\binom{m}{k}C(m,r)+lC(m,n))$-time algorithm, where $0 \leq l \leq k!\binom{n}{k}$ is related to similarities in $\mathcal{Q}$ and $C(\cdot,\cdot)$ is the quality measurement cost of a matching algorithm. A similar scheme can be achieved using the birthday paradox as well (sampling $O_\alpha(\sqrt{n})$ points in both sets repeatedly). Now, suppose that we are interested only by transformations $T \in \mathcal{T}$ that are meaningful for some application purposes. For example, those such that $|d(T(S_i),T(S_j)) - d(S_i,S_j)| \leq w$, where w depends on the input confidence. Then instead of testing all k-subsets of $\mathcal{Q}$ for a given k-tuple $(S_1,...,S_k)$. We first choose a point $Q_1 \in \mathcal{Q}$ and select as potential candidates for Q_2 only the points inside the ring of center Q_1, inner radius $d(S_1,S_2)-w$ and width $2w$ for some prescribed parameter w. We proceed iteratively by querying the set $\mathcal{Q}$ with geometric ranges reflecting the constraints on transformation T. Let $0 \leq t \leq k!\binom{n}{k}$ denotes the number of k-tuples of $\mathcal{Q}$ that satisfies the constraints. Quantity t depends on the self-similarity of point set $\mathcal{Q}$. For example, let us take $k=2$. Given a random drawn pair $\{S_1,S_2\}$ of set $\mathcal{S}$, we ask for all pairs $\{Q_1,Q_2\}$ of $\mathcal{Q}$ such that $|d(Q_1,Q_2)-d(S_1,S_2)| \leq w$. Ideally, we would like to enumerate the pairs of points of $\mathcal{Q}$ that have distance between $d(S_1,S_2)-w$ and $d(S_1,S_2)+w$. This is in itself a challenging problem. We rather proceed as follows: take turn in turn the points of $\mathcal{Q}$. For a current point Q_i, draw a ring of inner radius $d(S_1,S_2)-w$ and with $2w$. Then report all points of $\mathcal{Q}$ inside that geometric range. Let t be the number of points reported on all stages. If $w=0$, then t is also bounded by the number of times a distance repeat itself in a complete geometric graph (or the maximum number of incidences of m circles with m points, that is proven to be $O(m^{\frac{4}{3}}) = o(m^2)$ and strongly conjectured to be $O(n^{1+\epsilon})$ by Erdős). Building a data-structure based on that of Gupta et al. [5] we can report the t pairs to test in $O(nt^{\frac{2}{3}+\epsilon})$. This approach can be generalized in any dimension but we note that as soon as $d \geq 4$ the number of unit distances can be quadratic (for $d=3$ it is $O(n^{\frac{3}{2}+\epsilon})$). We can also use the scheme of queries for checking if a given transformation T is matching or not by first checking that it is for $O(1)$ points and then extending possibly to the all set. *Geometric filtering* in alignment method is a general approach that has to be tailored on a case-based study.

3 Applications to Mosaicing.

Given two images taken from a "geometric camera" whose optical center is fixed, there exists a homography (defined by the correspondence of four points, $k=4$ in nondegenerate position) that match the points of one image to the points of the other image. The figure describes the current implemented system [6].



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