Jensen Divergence Based SPD Matrix Means and Applications

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$\longrightarrow$ Many symmetric positive definite (SPD) matrix data sets (covariances, DTI, etc.)

Matrix mean defined as the average divergence minimizer:

$$\bar{M}_\alpha = \arg\min_{X \in \mathrm{Sym}^*_+} \frac{1}{n}\sum_{i=1}^n J_{LD}^{(\alpha)}(X, M_i)$$

The α-log det divergence: $J_{LD}^{(\alpha)}(X,Y) = \frac{4}{1-\alpha^2}\left(\frac{1-\alpha}{2}F(X) + \frac{1+\alpha}{2}F(Y) - F\left(\frac{1-\alpha}{2}X + \frac{1+\alpha}{2}Y\right)\right)$

$$F(X) = -\log\det X, \text{ convex functional}$$

The symmetrized α-log det divergence: $sJ_{LD}^{(\alpha)}(X,Y) = \frac{1}{2}\left(J_{LD}^{(\alpha)}(X,Y) + J_{LD}^{(\alpha)}(Y,X)\right).$

Invariant under inversion and invertible transformations:

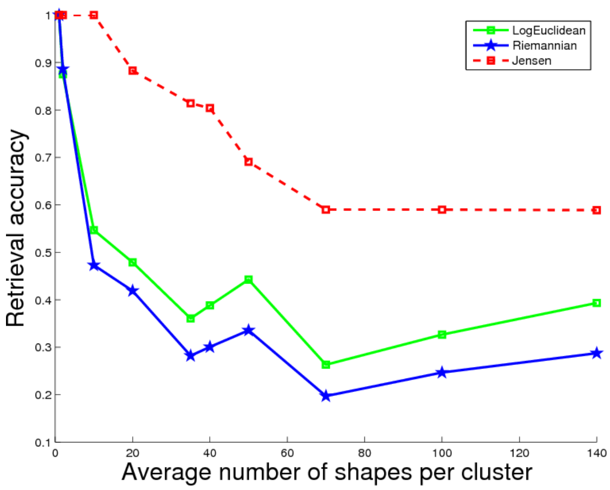
- $J_{LD}^{(\alpha)}(X,Y) = J_{LD}^{(\alpha)}(X^{-1}, Y^{-1})$,
- $J_{LD}^{(\alpha)}(CXC^T, CYC^T) = J_{LD}^{(\alpha)}(X,Y), \forall C \in \mathrm{GL}(d)$

Solve the minimization problem using the Convex-ConCave Procedure (CCCP):

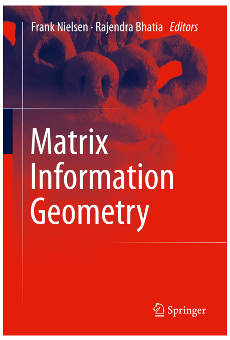
$C_0 = \frac{1}{n}\sum_{i=1}^n M_i$ For log-det divergence, $\nabla F(X) = \nabla F^{-1}(X) = -X^{-1}$.

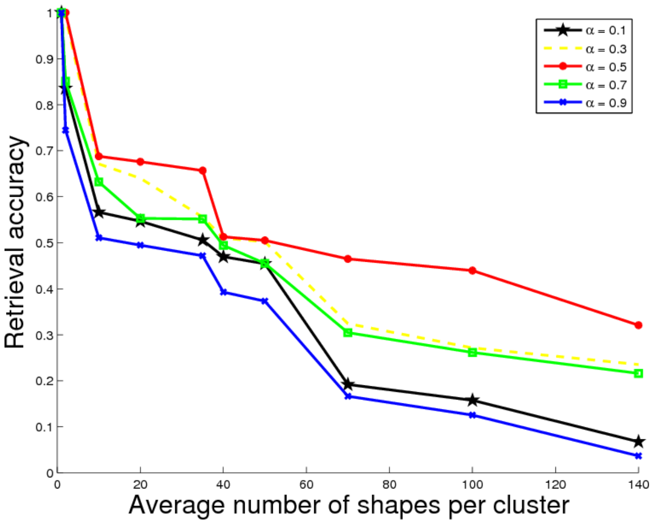
$$C_{t+1} = (\nabla F)^{-1}\left(\sum_{i=1}^n \frac{1}{n}(1-\alpha)\nabla F(\alpha M_i + (1-\alpha)C_t) + \alpha \nabla F(\alpha C_t + (1-\alpha)M_i)\right)$$

Experiments using synthetic data sets and shape clustering application:



Shape clustering using Riemannian, Log-Euclidean, and Jensen log-det centroids.





Impact of various α values